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Soft-gluon resummation for bottom fragmentation in top quark decay

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Abstract: We study soft-gluon radiation in top quark decay within the framework of perturbative fragmentation functions. We present results for the b-quark energy distribution, accounting for soft-gluon resummation to next-to-leading logarithmic accuracy in both the $\overline{\rm MS}$ coefficient function and in the initial condition of the perturbative fragmentation function. The results show a remarkable improvement and the b-quark energy spectrum in top quark decay exhibits very little dependence on factorization and renormalization scales. We present some hadron-level results in both x_B and moment space by including non-perturbative information determined from e^+e^- data.

Keywords: Heavy Quarks Physics, QCD, NLO Computations.

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Contents

1. Introduction

Heavy-flavour and in particular top quark physics is presently one of the main fields of investigation in theoretical and experimental particle physics. The current experiments at the Tevatron accelerator and, ultimately, at LHC [1] and e^+e^- Linear Colliders [2] will produce large amounts of top quark pairs, which will allow one to perform improved measurements of top properties, such as its mass. For this purpose, precise calculations for top production and decay processes are mandatory.

While fixed-order calculations reliably predict total cross sections or widths, differential distributions typically contain large logarithms associated with soft or collinear parton radiation. For heavy-quark production processes, although the quark mass m (much larger than the QCD scale Λ) acts as a regulator for the collinear singularity, event shapes still contain large $\alpha_S^n \ln^p(Q^2/m^2)$ (with $p \leq n$) terms, Q being a typical scale of the process, which make fixed-order predictions unreliable when $Q \gg m$. Such logarithms can be resummed by using the perturbative fragmentation approach [3], which factorizes the rate of heavy-quark production into the convolution of a coefficient function, describing the emission of a massless parton, and a perturbative fragmentation function $D(\mu_F, m)$, where μ_F is the factorization scale. The perturbative fragmentation function expresses the transition of the massless parton into the massive quark, and its value at any scale μ_F can be obtained by solving the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [4, 5] once an initial condition at a scale μ_{0F} is given. In ref. [3] the large $\ln(Q^2/m^2)$ collinear logarithms were resummed to next-to-leading logarithmic (NLL) accuracy in the case of heavy quark production in e^+e^- collisions and an explicit next-to-leading order (NLO) expression for $D(\mu_F, m)$, which was argued to be process independent, was given. More recently, ref. [6] has established the process independence in a more general way.

The approach of perturbative fragmentation has been extensively used for e^+e^- annihilation [6, 7, 8, 9, 10], hadron collisions [11, 12], photoproduction [13, 8] and, more recently, for bottom quark production in top quark decay $t \to bW$ [14].

The initial condition of the perturbative fragmentation function and the coefficient function, though free of the collinear large $\ln(Q^2/m^2)$, contain large logarithms which are due to soft-gluon radiation. The ones contained in the initial condition are process independent [6], and were already included in the top-to-bottom decay process in [14], where the complete $\mathcal{O}(\alpha_S)$ calculation of the $t \to bW(g)$ process was also performed. The large logarithms contained in the coefficient function are instead process dependent, and have to be evaluated for every specific case. It is precisely the purpose of this paper to extend the analysis of ref. [14] and to present results for soft-gluon resummation in the coefficient function. This will allow, together with the results of [6], to complete the evaluation of soft-gluon effects to NLL accuracy for the top-to-bottom decay process, and to investigate the impact on the b-quark energy distribution.

After the fragmentation of heavy quarks in e^+e^- collisions considered in [6], this is the first process whose large logarithms (both collinear and soft) are fully resummed to NLL accuracy within the perturbative fragmentation function formalism. The consistency of this perturbative description with the one used in the e^+e^- process makes it possible to fit non-perturbative information from e^+e^- data and use it to make predictions. We shall therefore be able to predict the spectrum for b-flavoured hadron energy distributions in top decay using e^+e^- experimental data from LEP.

The outline of the paper is as follows. In section 2 we review bottom quark production in top quark decay within the framework of perturbative fragmentation. In section 3 we present analytic results for the NLL soft-gluon resummation of the coefficient function in top decay. At the end of the section we also comment on the relation between our results and previous work on soft-gluon resummation in heavy-flavour decay [15, 16, 17, 18]. In section 4 we show the b-quark energy distribution in top decay and investigate the impact of soft-gluon resummation. In section 5 we discuss inclusion of non-perturbative effects and present results for b-flavoured hadron spectra in top decay. In section 6 we summarize our main results.

2. Perturbative fragmentation and top quark decay

We consider top decay into a bottom quark and a real W boson plus, to order α_S, a gluon:

$$t(p_t) \to b(p_b)W(p_W)\,(g(p_g)) \tag{2.1}$$

and define the bottom and gluon normalized energy fractions x_b and x_g:

$$x_b = \frac{1}{1-w}\frac{2p_b\cdot p_t}{m_t^2}\,, \qquad x_g = \frac{1}{1-w}\frac{2p_g\cdot p_t}{m_t^2}\,, \tag{2.2}$$

where $w = m_W^2/m_t^2$. Neglecting the b mass, we have $0 \le x_{b,g} \le 1$.

Order α_S corrections to the decay process (2.1) were considered in [14]. It was observed there that, since $m_b \ll m_t$, one can readily neglect m_b/m_t power suppressed terms, but on the other hand it is important to resum to all orders terms enhanced, at order α_S, by the presence of $\ln(m_t^2/m_b^2)$. Such a resummation was performed in [14] by employing the perturbative fragmentation formalism [3, 6]: The differential width for the production of a massive b quark in top decay is written in terms of the convolution

$$\frac{1}{\Gamma_B}\frac{d\Gamma}{dx_b}(x_b,m_t,m_W,m_b) = \sum_i \int_{x_b}^1 \frac{dz}{z}\left[\frac{1}{\Gamma_B}\frac{d\hat{\Gamma}_i}{dz}(z,m_t,m_W,\mu_F)\right]^{\overline{\rm MS}} D_i\left(\frac{x_b}{z},\mu_F,m_b\right) + \\ +\mathcal{O}\left(\left(\frac{m_b}{m_t}\right)^p\right), \tag{2.3}$$

where Γ_B is the width of the Born process $t \to bW$, $d\hat{\Gamma}_i/dz$ is the differential width for the production of a massless parton i in top decay with energy fraction z, $D_i(x,\mu_F,m_b)$ is the perturbative fragmentation function for a parton i to fragment into a massive b quark, μ_F is the factorization scale. The term $\mathcal{O}\left((m_b/m_t)^p\right)$ on the right-hand side stands for contributions that are suppressed by some power p ($p \geq 1$) of m_b in the $m_b \ll m_t$ regime. Of course, non-perturbative corrections of the type Λ/m_t and Λ/m_b are understood on the right-hand side of eq. (2.3). We shall use everywhere a branching fraction $B(t \to bW) = 1$, and only include $i = b$ in the above summation.

The massless differential distribution $(1/\Gamma_B)\, d\hat{\Gamma}_i/dz$ (which is what we shall also refer to as "coefficient function") is defined in the $\overline{\rm MS}$ factorization scheme after subtraction of the collinear singularities. It has been calculated at order α_S in [14]. In the following we shall often use its Mellin moments, defined by

$$\hat{\Gamma}_N = \int_0^1 dz\, z^{N-1}\frac{1}{\Gamma_B}\frac{d\hat{\Gamma}}{dz}(z)\,. \tag{2.4}$$

In moment space the convolution (2.3) can then be rewritten as

$$\Gamma_N(m_t,m_W,m_b) = \hat{\Gamma}_N(m_t,m_W,\mu_F)D_{b,N}(\mu_F,m_b)\,. \tag{2.5}$$

The perturbative fragmentation function $D_b(x,\mu_F,m_b)$ at any scale μ_F can be obtained by solving the DGLAP equations. As shown in [6], as long as one can neglect contributions proportional to powers of $(m_b/m_t)^p$, the initial condition for the perturbative fragmentation function, which we evaluate at a scale μ_{0F}, is process independent (but scheme dependent). In the $\overline{\rm MS}$ scheme it reads [3]:

$$D_b^{\rm ini}(x,\mu_{0F},m_b) = \delta(1-x) + \frac{\alpha_S(\mu_0^2)C_F}{2\pi}\left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_{0F}^2}{m_b^2} - 2\ln(1-x) - 1\right)\right]_+ . \tag{2.6}$$

The solution of the DGLAP equations in the non-singlet sector, for the evolution from the scale μ_{0F} to μ_F, is given in moment space by:

$$D_{b,N}(\mu_F,m_b) = D_{b,N}^{\rm ini}(\mu_{0F},m_b)\exp\left\{\frac{P_N^{(0)}}{2\pi b_0}\ln\frac{\alpha_S(\mu_{0F}^2)}{\alpha_S(\mu_F^2)} + \right. \\ \left. + \frac{\alpha_S(\mu_{0F}^2)-\alpha_S(\mu_F^2)}{4\pi^2 b_0}\left[P_N^{(1)} - \frac{2\pi b_1}{b_0}P_N^{(0)}\right]\right\}, \tag{2.7}$$

In eq. (2.7) $P_N^{(0)}$ and $P_N^{(1)}$ are the Mellin transforms of the leading and next-to-leading order Altarelli-Parisi splitting vertices, and their explicit expression can be found, e.g., in [3]. b_0 and b_1 are the first two coefficients of the QCD β-function

$$b_0 = \frac{33 - 2n_f}{12\pi}, \qquad b_1 = \frac{153 - 19n_f}{24\pi^2}, \tag{2.8}$$

which enter the following expression for the strong coupling constant at a scale Q^2:

$$\alpha_S(Q^2) = \frac{1}{b_0 \ln(Q^2/\Lambda^2)} \left\{ 1 - \frac{b_1 \ln\left[\ln(Q^2/\Lambda^2)\right]}{b_0^2 \ln(Q^2/\Lambda^2)} \right\}. \tag{2.9}$$

Equation (2.7) resums to all order terms containing large $\ln(\mu_F^2/\mu_{0F}^2)$. In particular, leading $(\alpha_S^n \ln^n(\mu_F^2/\mu_{0F}^2))$ and next-to-leading $(\alpha_S^n \ln^{n-1}(\mu_F^2/\mu_{0F}^2))$ logarithms are resummed. Setting, as done in [14], $\mu_F \simeq m_t$ and $\mu_{0F} \simeq m_b$, one resums the large $\ln(m_t^2/m_b^2)$ terms with NLL accuracy. These are indeed the large collinear logarithms exhibited by the fixed-order calculation with a massive b quark [14].

3. Soft-gluon resummation

In this section we address the problem of soft-gluon resummation in top quark decay. The $\overline{\rm MS}$ coefficient function computed in [14] and the initial condition of the perturbative fragmentation function (2.6) contain terms behaving like $1/(1-x)_+$ or $[\ln(1-x)/(1-x)]_+$, which become arbitrarily large when x approaches one. This is equivalent to contributions proportional to $\ln N$ and $\ln^2 N$ in moment space, as can be seen by writing the $\overline{\rm MS}$ coefficient function [14] in the large-N limit:[1]

$$\begin{aligned}\hat{\Gamma}_N(m_t, m_W, \mu_F) = 1 + \frac{\alpha_S C_F}{2\pi} \Bigg\{ & 2\ln^2 N + \left[4\gamma_E + 2 - 4\ln(1-w) - 2\ln\frac{m_t^2}{\mu_F^2} \right] \ln N + \\ & + K(m_t, m_W, \mu_F) + \mathcal{O}\left(\frac{1}{N}\right) \Bigg\}\end{aligned} \tag{3.1}$$

where $\gamma_E = 0.577\ldots$ is the Euler constant and $w = m_W^2/m_t^2$, as defined in section 1. In eq. (3.1) we have introduced the function $K(m_t, m_W, \mu_F)$, which contains terms which are constant with respect to N. It reads:

$$\begin{aligned}K(m_t, m_W, \mu_F) = & \left(\frac{3}{2} - 2\gamma_E\right) \ln\frac{m_t^2}{\mu_F^2} + 2\gamma_E^2 + 2\gamma_E\left[1 - 2\ln(1-w)\right] + \\ & + 2\ln w \ln(1-w) - 2\frac{1-w}{1+2w}\ln(1-w) - \frac{2w}{1-w}\ln w + \\ & + 4\mathrm{Li}_2(1-w) - 6 - \frac{\pi^2}{3}.\end{aligned} \tag{3.2}$$

The $x \to 1$ $(N \to \infty)$ limit corresponds to soft-gluon radiation in top decay. These soft logarithms need to be resummed to all orders in α_S [19, 20] to improve our prediction.

[1] Following [18], we note that, by defining $n = N\exp(\gamma_E)$, we could rewrite this expression in terms of $\ln(n)$ rather than $\ln N$, with no γ_E terms explicitly appearing.

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Soft logarithms in the initial condition of the perturbative fragmentation function are process independent. We can hence resum them with NLL accuracy using the result presented in [6], which we do not report here for the sake of brevity. We present instead the results for the NLL resummation of process-dependent soft-gluon contributions in the $\overline{\rm MS}$ coefficient function.

In order to resum the large terms in eq. (3.1), we follow standard techniques [19], evaluate the amplitude of the process in eq. (2.1) at $\mathcal{O}(\alpha_S)$ in the eikonal approximation and exponentiate the result. The eikonal current reads:

$$|J(p_t, p_b, p_g)|^2 = \left| \frac{m_t^2}{(p_t \cdot p_g)^2} - 2\frac{(p_t \cdot p_b)}{(p_t \cdot p_g)(p_b \cdot p_g)} \right| . \tag{3.3}$$

For the sake of comparison with [19], we express the $\mathcal{O}(\alpha_S)$ width in the soft approximation as an integral over the variables[2] $q^2 = (p_b + p_g)^2 x_g$ and $z = 1 - x_g$, with $0 \le q^2 \le m_t^2(1-w)^2(1-z)^2$ and $0 \le z \le 1$. The limits $z \to 1$ and $q^2 \to 0$ correspond to soft and collinear emission respectively. In soft approximation, $z \simeq x_b$, the b-quark energy fraction. We obtain:

$$\hat{\Gamma}_N(m_t, m_W, \mu_F) = \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left[\int_{\mu_F^2}^{m_t^2(1-w)^2(1-z)^2} \frac{dq^2}{q^2} \alpha_S - \right.$$
$$\left. - \frac{1}{m_t^2(1-w)^2(1-z)^2} \int_0^{m_t^2(1-w)^2(1-z)^2} dq^2 \alpha_S \right] . \tag{3.4}$$

In eq. (3.4) we have regularized the collinear singularity setting the cutoff $q^2 \ge \mu_F^2$. At NLL accuracy level, this is equivalent to $\overline{\rm MS}$ subtraction in dimensional regularization [6, 21].

In order to perform soft-gluon resummation to NLL accuracy a number of operations have to be performed on this expression. We set the argument of α_S equal to q^2 and, as far as the collinear-divergent term is concerned, we perform the replacement

$$\frac{C_F}{\pi} \frac{\alpha_S(q^2)}{q^2} \to \frac{A\left[\alpha_S(q^2)\right]}{q^2} , \tag{3.5}$$

where the function $A(\alpha_S)$ was introduced in [19] and is detailed below. Moreover, the integral over q^2 of the non-collinear divergent term can be written, up to terms beyond NLL accuracy, as

$$\frac{1}{m_t^2(1-w)^2(1-z)^2} \int_0^{m_t^2(1-w)^2(1-z)^2} dq^2 \alpha_S(q^2) = \alpha_S\left(m_t^2(1-w)^2(1-z)^2\right) . \tag{3.6}$$

This term describes soft radiation at large-angle, i.e. not collinear enhanced, and it is characteristic of processes where a heavy quark (the top quark in our case, the bottom

[2] We point out that our definition of the integration variable q^2 is analogous to the quantity $(1-z)k^2$ to which the authors of ref. [19] set the scale for α_S for soft-gluon resummation in Drell-Yan and Deep-Inelastic-Scattering processes. For small-angle radiation, $q^2 \simeq q_T^2$, the gluon transverse momentum with respect to the b-quark line. The variable z is analogous to $z = 1 - E_g/E_q$ of ref. [19].

quark in [15, 16, 17, 18]) is present. It can be generalized to all orders by replacing eq. (3.6) according to:

$$-\frac{C_F}{\pi}\alpha_S\left(m_t^2(1-w)^2(1-z)^2\right) \to S\left[\alpha_S\left(m_t^2(1-w)^2(1-z)^2\right)\right] . \tag{3.7}$$

This function is called $\Gamma(\alpha_S)$ in [15], $B(\alpha_S)$ in [16], $S(\alpha_S)$ in [17], $D(\alpha_S)$ in [18]. We now insert eqs. (3.5-3.7) into eq. (3.4) and exponentiate the result. We obtain:

$$\begin{aligned}\ln\Delta_N = & \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left\{\int_{\mu_F^2}^{m_t^2(1-w)^2(1-z)^2}\frac{dq^2}{q^2}A\left[\alpha_S(q^2)\right]+\right.\\ & \left. + S\left[\alpha_S\left(m_t^2(1-w)^2(1-z)^2\right)\right]\right\} .\end{aligned} \tag{3.8}$$

We would like to evaluate eq. (3.8) to NLL level. The function $A(\alpha_S)$ can be expanded as follows:

$$A(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A^{(n)} . \tag{3.9}$$

The first two coefficients are needed at NLL level and are given by [19, 22]:

$$A^{(1)} = C_F , \tag{3.10}$$

$$A^{(2)} = \frac{1}{2}C_F\left[C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5}{9}n_f\right] , \tag{3.11}$$

where $C_F = 4/3$, $C_A = 3$ and n_f is the number of quark flavours, which we shall take equal to five for bottom production.

The function $S(\alpha_S)$ can be expanded according to:

$$S(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n S^{(n)} . \tag{3.12}$$

At NLL level, we are just interested in the first term of the above expansion, which is given by:

$$S^{(1)} = -C_F . \tag{3.13}$$

The integral in eq. (3.8) can be performed, up to NLL accuracy, by making the following replacement [19]:

$$z^{N-1}-1 \to -\Theta\left(1-\frac{e^{-\gamma_E}}{N}-z\right) , \tag{3.14}$$

Θ being the Heaviside step function. This leads to writing the following result for the function Δ_N:

$$\Delta_N(m_t, m_W, \alpha_S(\mu^2), \mu, \mu_F) = \exp\left[\ln N g^{(1)}(\lambda) + g^{(2)}(\lambda, \mu, \mu_F)\right] , \tag{3.15}$$

with

$$\lambda = b_0\alpha_S(\mu^2)\ln N , \tag{3.16}$$

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and the functions $g^{(1)}$ and $g^{(2)}$ given by

$$g^{(1)}(\lambda) = \frac{A^{(1)}}{2\pi b_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]\,, \tag{3.17}$$

$$\begin{aligned} g^{(2)}(\lambda,\mu,\mu_F) = & \frac{A^{(1)}}{2\pi b_0}\left[\ln\frac{m_t^2(1-w)^2}{\mu_F^2} - 2\gamma_E\right]\ln(1-2\lambda) + \\ & + \frac{A^{(1)}b_1}{4\pi b_0^3}\left[4\lambda + 2\ln(1-2\lambda) + \ln^2(1-2\lambda)\right] - \\ & - \frac{1}{2\pi b_0}[2\lambda + \ln(1-2\lambda)]\left(\frac{A^{(2)}}{\pi b_0} + A^{(1)}\ln\frac{\mu^2}{\mu_F^2}\right) + \\ & + \frac{S^{(1)}}{2\pi b_0}\ln(1-2\lambda)\,. \end{aligned} \tag{3.18}$$

In eq. (3.15) the term $\ln N g^{(1)}(\lambda)$ accounts for the resummation of leading logarithms $\alpha_S^n \ln^{n+1} N$ in the Sudakov exponent, while the function $g^{(2)}(\lambda,\mu,\mu_F)$ resums NLL terms $\alpha_S^n \ln^n N$.

Furthermore, we follow ref. [6] and in our final Sudakov-resummed coefficient function we also include the constant terms of eq. (3.2):

$$\begin{aligned} \hat{\Gamma}_N^S(m_t,m_W,\alpha_S(\mu^2),\mu,\mu_F) = & \left[1 + \frac{\alpha_S(\mu^2)\,C_F}{2\pi}K(m_t,m_W,\mu_F)\right]\times \\ & \times\exp\left[\ln N g^{(1)}(\lambda) + g^{(2)}(\lambda,\mu,\mu_F)\right]. \end{aligned} \tag{3.19}$$

One can check that the $\mathcal{O}(\alpha_S)$ expansion of eq. (3.19) yields eq. (3.1).

We now match the resummed coefficient function to the exact first-order result, so that also $1/N$ suppressed terms, which are important in the region $x_b < 1$, are taken into account. We adopt the same matching prescription as in [6]: we add the resummed result to the exact coefficient function and, in order to avoid double counting, we subtract what they have in common, i.e. the up-to-$\mathcal{O}(\alpha_S)$ terms in the expansion of eq. (3.19). Our final result for the resummed coefficient function reads:[3]

$$\begin{aligned} \hat{\Gamma}_N^{\rm res}(m_t,m_W,\alpha_S(\mu^2),\mu,\mu_F) = & \hat{\Gamma}_N^S(m_t,m_W,\alpha_S(\mu^2),\mu,\mu_F) - \\ & - \left[\hat{\Gamma}_N^S(m_t,m_W,\alpha_S(\mu^2),\mu,\mu_F)\right]_{\alpha_S} + \\ & + \left[\hat{\Gamma}_N(m_t,m_W,\alpha_S(\mu^2),\mu,\mu_F)\right]_{\alpha_S}\,, \end{aligned} \tag{3.20}$$

where $[\hat{\Gamma}_N^S]_{\alpha_S}$ and $[\hat{\Gamma}_N]_{\alpha_S}$ are respectively the expansion of eq. (3.19) up to $\mathcal{O}(\alpha_S)$ and the full fixed-order top-decay coefficient function at $\mathcal{O}(\alpha_S)$, evaluated in appendix B of ref. [14].

Before closing this section we would like to add more comments on the comparison of our resummed expression with other similar results obtained in heavy quark decay processes [15, 16, 17, 18].

[3] Alternative ways of matching, identical up to order α_S and differing in higher-order, subleading terms, are of course possible.

Besides the obvious replacement of a bottom quark with a top in the initial state, our work presents other essential differences. We have resummed large collinear logarithms $\alpha_S \ln(m_t^2/m_b^2)$, while refs. [15, 16, 17, 18] just address the decay of heavy quarks into massless quarks. Moreover, this work still differs in a critical issue. Those papers are concerned with observing the lepton produced by the W decay or the photon in the $b \to X_s\gamma$ process, while we wish instead to observe the outgoing b quark. This is immediately clear from the choice of the x variable whose $x \to 1$ endpoint leads to the Sudakov logarithms. In our case it is the normalized energy fraction of the outgoing bottom quark; in [15, 16, 17, 18] it is instead related to the energy of either the lepton or the radiated photon.

The most evident effect of this different perspective is that an additional scale, namely the invariant mass of the recoiling hadronic jet, enters the results [15, 16, 17, 18], but it is instead absent in our case. An additional function (called $\gamma(\alpha_S)$ in [15, 16], $C(\alpha_S)$ in [17], $B(\alpha_S)$ in [18]) appears in those papers. The argument of α_S in this function is related to the invariant mass of the unobserved final state jet constituted by the outgoing quark and the gluon(s). It is worth noting that an identical function, called $B\left[\alpha_S\left(Q^2(1-z)\right)\right]$, also appears in the e^+e^- [6] and DIS [19] massless coefficient functions, where it is again associated with the invariant mass of the unobserved jet. We do not have instead any $B(\alpha_S)$ contribution in our result. In fact, this function contains collinear radiation associated with an undetected quark, which we do not have since the b quark is observed.

We also observe that to order α_s the coefficient $S^{(1)}$ coincides with the corresponding $H^{(1)}$ of the function $H\left[\alpha_S\left(m_b^2(1-z)^2\right)\right]$, which resums soft terms in the initial condition of the perturbative fragmentation function [6]. It will be very interesting to compare the functions $S(\alpha_S)$ and $H(\alpha_S)$ at higher orders as well.

One final comment we wish to make is that, as expected, in our final result, eq. (2.5), which accounts for NLL soft resummation in both the coefficient function and the initial condition of the perturbative fragmentation function, $\alpha_S^n \ln^{n+1} N$ terms do not appear, since they are due to soft *and* collinear radiation. Both the quarks being heavy, only the former leads to a logarithmic enhancement. Double logarithms are generated by a mismatch in the lower and upper q^2 integration limit over the $A[\alpha_S(q^2)]$ function in the exponent of the resummation expression. In our case both of them have the same functional dependence with respect to z, i.e. $(1-z)^2$ (see eq. (3.8) and eq. (69) of ref. [6]). The cancellation of the $\alpha_S^n \ln^{n+1} N$ term can be explicitly seen at order α_S by comparing the large-N limit for the coefficient function, eq. (3.1), and the initial condition (eq. (45) of ref. [6]): the $\ln^2 N$ terms have identical coefficients and opposite signs.

4. Energy spectrum of the b quark

In this section we present results for the b-quark energy distribution in top decay. The b-quark spectrum in N-space $\Gamma_N(m_t, m_W, \mu_F)$ is given by eq. (2.5). In the following we shall normalize Γ_N to the full NLO width Γ [23], so that $\Gamma_1 = 1$ will always hold. Results in x_b-space will be obtained by inverting numerically eq. (2.5) via contour integration in the complex plane, using the minimal prescription [24] to avoid the Landau pole.

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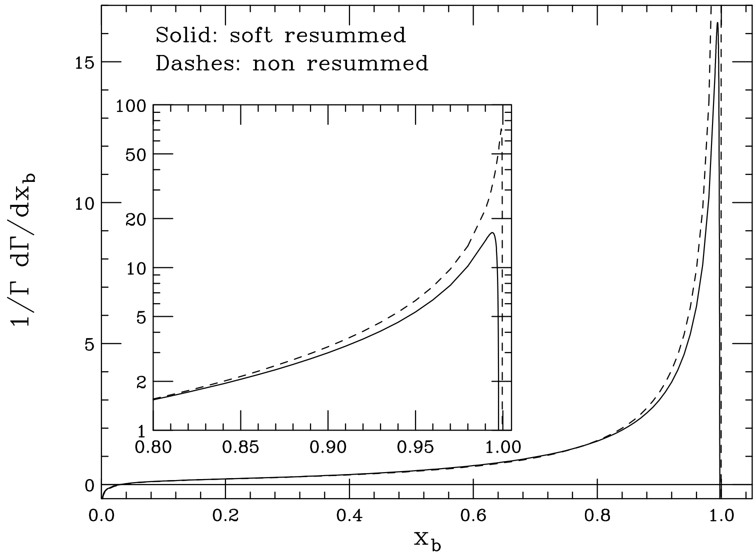

Figure 1: b-quark energy distribution in top decay according to the perturbative fragmentation approach, with (solid line) and without (dashes) NLL soft-gluon resummation. In the inset figure, we show the same curves on a logarithmic scale, for $x_b > 0.8$. We have set $\mu_F = \mu = m_t$ and $\mu_{0F} = \mu_0 = m_b$.

In order to estimate the effect of the NLL soft-gluon resummation, we compare our result with ref. [14] and use the same values for the parameters: $m_t = 175\,\mathrm{GeV}$, $m_b = 5\,\mathrm{GeV}$, $m_W = 80\,\mathrm{GeV}$ and $\Lambda^{(5)} = 200\,\mathrm{MeV}$.

In figure 1 we present the x_b distribution according to the approach of perturbative fragmentation, with and without NLL soft-gluon resummation. For the scales appearing in eqs. (2.6), (2.7) and (3.1) we have set $\mu_F = \mu = m_t$ and $\mu_0 = \mu_{0F} = m_b$. We note that the two distributions agree for $x_b \lesssim 0.8$, while for larger x_b values the resummation of large terms $x_b \to 1$ smoothens out the distribution, which exhibits the Sudakov peak. Both distributions become negative for $x_b \to 0$ and $x_b \to 1$. As discussed in [14], the negative behaviour at small x_b can be related to the presence of unresummed $\alpha_S \ln x_b$ terms in the coefficient function. At large x_b, we approach instead the non-perturbative region, and resumming leading and next-to-leading logarithms is still not sufficient to correctly describe the spectrum for x_b close to 1. In fact, the range of reliability of the perturbative calculation has been estimated to be $x_b \lesssim 1 - \Lambda/m_b \simeq 0.95$ [6].

It is interesting to investigate the dependence of phenomenological distributions on the renormalization and factorization scales which enter the coefficient function (μ and μ_F) and the initial condition of the perturbative fragmentation function (μ_0 and μ_{0F}). In particular, it is worth comparing the b-energy spectra with and without soft resummation. For the scales μ and μ_F we consider the values $m_t/2$, m_t and $2m_t$; for μ_{0F} and μ_0 the choices are $m_b/2$, m_b and $2m_b$. Figures 2 and 3 show the dependence of the x_b spectrum on the factorization scales μ_F and μ_{0F}; the dependence on the renormalization scales μ and μ_0 is exhibited in figures 4 and 5.

We note that all distributions which include soft-gluon resummation exhibit a reduced dependence on the factorization and renormalization scales.

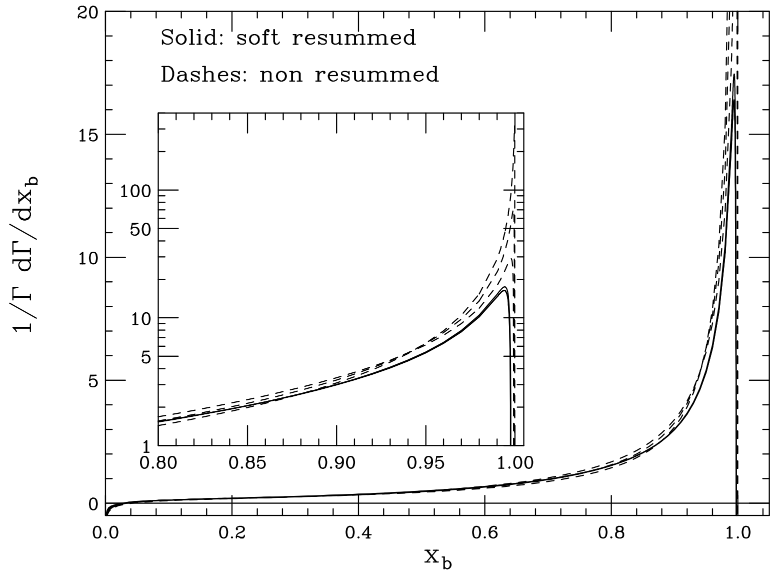

Figure 2: b-quark energy spectrum for different values of the factorization scale μ_F, with (solid) and without (dashes) NLL soft-gluon resummation. The other scales are fixed at $\mu = m_t$, $\mu_0 = \mu_{0F} = m_b$. As in figure 1, in the inset figure, we present the same plots for large values of x_b, on a logarithmic scale.

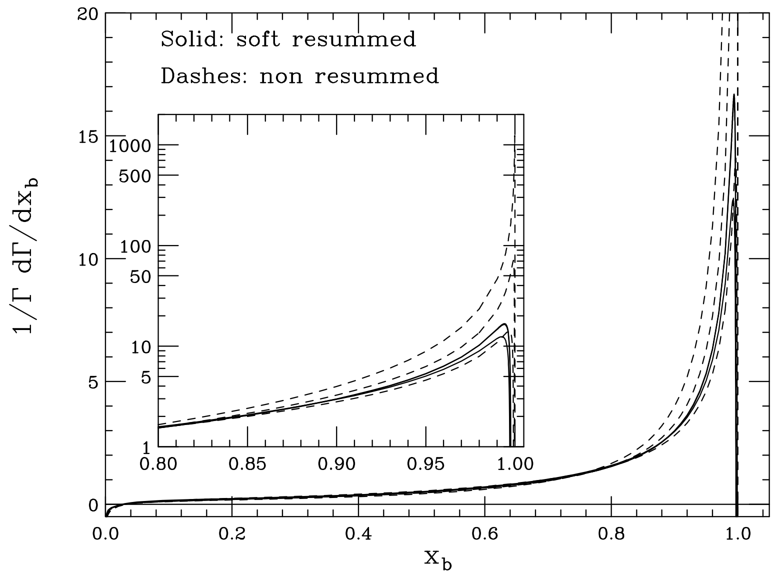

Figure 3: As in figure 2, but for different values of μ_{0F}. The other scales are fixed at $\mu = \mu_F = m_t$, $\mu_0 = m_b$.

Figure 2 shows that curves obtained using different values of μ_F are almost indistinguishable once soft resummation is included; the unresummed plots exhibit instead a stronger effect of the chosen value for μ_F. A similar result also holds for the scale μ_{0F}: the dependence of the plots on its actual value for $x_b > 0.8$ is small if soft logarithms are resummed and quite strong if the prediction is unresummed (figure 3).

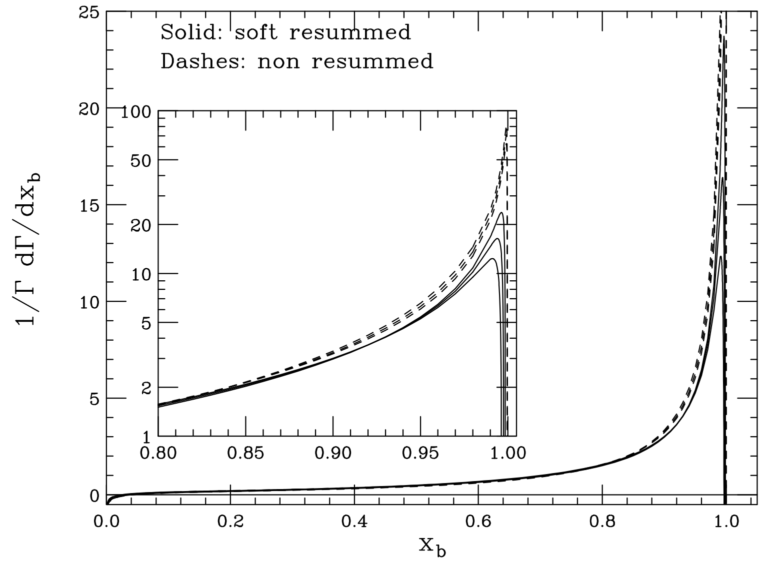

Figure 4: As in figure 2, but for different values of the renormalization scale μ. The other scales are fixed at $\mu_F = m_t$, $\mu_0 = \mu_{0F} = m_b$.

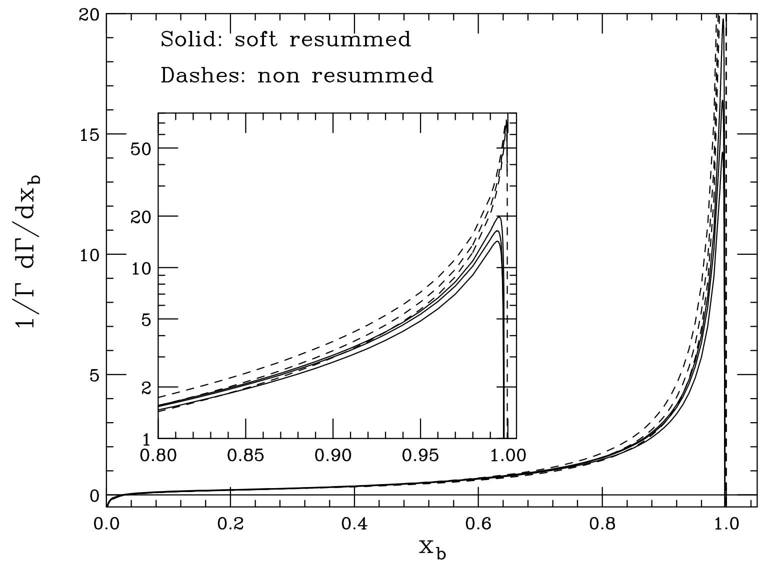

Figure 5: As in figure 2, but for different values of the renormalization scale μ_0. The other scales are fixed at $\mu = \mu_F = m_t$, $\mu_{0F} = m_b$.

The choice of the value for the renormalization scale μ appearing in eq. (3.1) affects only the neighbourhood of the Sudakov peak of the resummed predictions, at x_b-values very close to one (figure 4), where, as we have pointed out, our perturbative approach is anyway unreliable. The effect of the choice of the renormalization scale μ_0 on the soft-resummed spectra is slightly larger than the one of μ and visible at $x_b < 1$ as well (figure 5). As for the non-soft-resummed predictions, although all dashed curves in figures 4 and 5 seem to converge to same point for $x_b \to 1$, the overall dependence on μ and μ_0 for $x_b < 1$ is stronger than for the resummed predictions.

As a whole, one can say that the implementation of NLL soft-gluon resummation, along with the NLL DGLAP evolution for the perturbative fragmentation function, yields a remarkable improvement of our phenomenological results, since the reduced dependence on the choice of factorization and renormalization scales in the region where the perturbative approach is reliable corresponds to a reduction of the theoretical uncertainty.

5. Energy spectrum of b-flavoured hadrons in top decay

In this section we consider the problem of including a non-perturbative component on top of the perturbative result, so as to make predictions for observable b-flavoured hadrons (like B mesons) in top decay. At the same time, we also account for the inclusion of NLL soft and collinear resummation.

We write the normalized rate for the production of b-hadrons B as a convolution of the rate for the production of b quarks in top decay, given by eq. (2.5), and a non perturbative fragmentation function $D^{np}(x)$:

$$\frac{1}{\Gamma}\frac{d\Gamma^B}{dx_B}(x_B,m_t,m_W,m_b)=\frac{1}{\Gamma}\int_{x_B}^1\frac{dz}{z}\frac{d\Gamma^b}{dz}(z,m_t,m_W,m_b)D^{np}\left(\frac{x_B}{z}\right), \tag{5.1}$$

where x_B is the B normalized energy fraction:

$$x_B=\frac{1}{1-w}\frac{2p_B\cdot p_t}{m_t^2}\,, \tag{5.2}$$

p_B being the B four-momentum. Since $D^{np}(x)$ contains non-perturbative information, it cannot — for the time being — be calculated from first principles in QCD, but can only be extracted from data. We shall assume a universality property for such a function, and extract it from fits to B-production data collected at LEP in e^+e^- collisions. In particular, we can choose different functional forms for $D^{np}(x)$, and tune these hadronization models to the data. We shall consider a power law with two tunable parameters:

$$D^{np}(x;\alpha,\beta)=\frac{1}{B(\beta+1,\alpha+1)}(1-x)^\alpha x^\beta\,, \tag{5.3}$$

the model of Kartvelishvili et al. [30]:[4]

$$D^{np}(x;\delta)=(1+\delta)(2+\delta)(1-x)x^\delta\,, \tag{5.4}$$

and the Peterson et al. model [31]:

$$D^{np}(x;\epsilon)=\frac{A}{x[1-1/x-\epsilon/(1-x)]^2}\,. \tag{5.5}$$

In eq. (5.3), $B(x,y)$ is the Euler Beta function; in (5.5) A is a normalization constant.

[4]We correct a typing mistake of ref. [14], where the normalization factor is the inverse of the correct one. The numerical results of ref. [14] are nonetheless obtained using the correct normalization of the Kartvelishvili non-perturbative fragmentation function.

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In order for our procedure to be self-consistent, care must be taken to employ the same underlying perturbative description in both the $e^+e^- \to b\bar{b}$ process (where the non-perturbative contribution is fitted) and the $t \to bW$ one (where it is used). This will be ensured by using in both cases NLO, $\overline{\mathrm{MS}}$ coefficient functions, along with a fully NLL soft-gluon resummed description, with the large collinear logarithms resummed to NLL accuracy by DGLAP evolution. For the coefficient functions in e^+e^- annihilation we shall refer to [25].

Fits to data points can be performed either in x_B-space, or, as recently advocated [10], in the conjugated moment space. When fitting in x_B space we discard data points close to $x_B = 0$ and $x_B = 1$ and consider ALEPH [26] data[5] in the range $0.18 \lesssim x_B \lesssim 0.94$. The results of our fits are shown in table 1.

α	0.51 ± 0.15
β	13.35 ± 1.46
$\chi^2(\alpha,\beta)/\mathrm{dof}$	2.56/14
δ	17.76 ± 0.62
$\chi^2(\delta)/\mathrm{dof}$	10.54/15
ϵ	$(1.77 \pm 0.16) \times 10^{-3}$
$\chi^2(\epsilon)/\mathrm{dof}$	29.83/15

Table 1: Results of fits to $e^+e^- \to b\bar{b}$ ALEPH data, using matched coefficient function and initial condition, with NLL DGLAP evolution and NLL soft-gluon resummation. We set $\Lambda^{(5)} = 200\,\mathrm{MeV}$, $\mu_{0F} = \mu_0 = m_b = 5\,\mathrm{GeV}$ and $\mu_F = \mu = \sqrt{s} = 91.2\,\mathrm{GeV}$. α and β are the parameters in the power law (5.3), δ refers to (5.4), ϵ to (5.5). The fits have been performed neglecting the correlations between the data points.

The best-fit values for the parameters of the hadronization models are quite different from the ones quoted in [14], where soft-gluon resummation in the e^+e^- coefficient function has not been used in the fits. The models in eqs. (5.3) and (5.4) yield very good fits to the data,[6] as already found in [14], while the model (5.5) is marginally consistent.

Using the results in table 1 we can give predictions for the spectrum of b-flavoured hadrons in top decay. As in ref. [14], to account for the errors on the best-fit parameters, we shall plot bands which correspond to predictions at one-standard-deviation confidence level. In figure 6 we show our predictions for the x_B distribution using the three models fitted to the ALEPH data. At one-standard-deviation confidence level, the three predictions are different, with the Peterson model yielding a distribution which lies quite far from the other two and peaked at larger x_B-values. Within two standard deviations, the predictions obtained using the models (5.3) and (5.4) are nonetheless in agreement.[7] We also note that figure 6 is qualitatively similar to figure 4 of ref. [14], where soft-gluon resummation in the coefficient function had not been implemented. In fact, the different perturbative content of these curves is now compensated by different values for the

[5] Good-quality data are also available from SLD [27] on b-flavoured mesons and baryons. While this paper was being finalized, the OPAL [28] and DELPHI [29] Collaborations also published new results, which are fully compatible with the ones from ALEPH.

[6] We have also fitted the SLD data and found qualitatively similar results. However, the value for the parameters which best fit the SLD data are different from the ones obtained for ALEPH and quoted in table 1.

[7] The differences between the various models mainly originate from the varying quality of the fits to e^+e^- data where, as the χ^2 values in table 1 seem to suggest, a given model is sometimes not really able to describe the data properly, due to its too restrictive functional form.

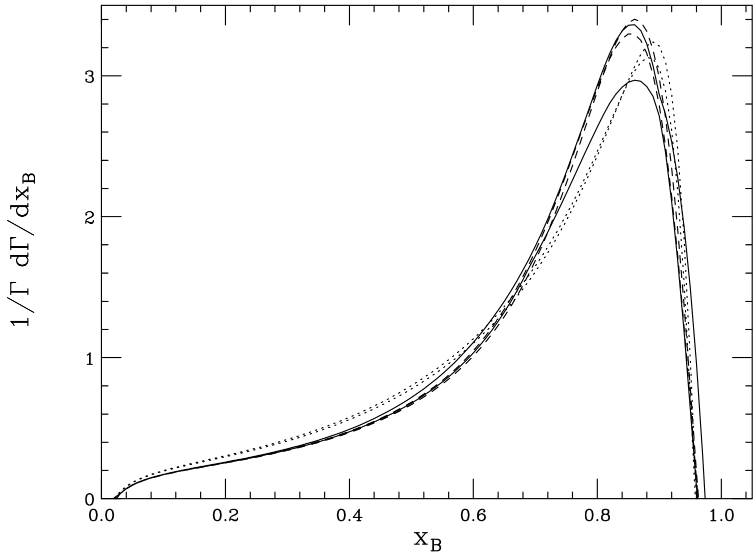

Figure 6: x_B spectrum in top decay, with the hadronization modeled according to a power law (solid lines), the Kartvelishvili et al. (dashes) and the Peterson (dots) model, with the relevant parameters fitted to the ALEPH data. The plotted curves are the edges of bands at one-standard-deviation confidence level. NLL soft-gluon resummation is included. We set $\mu_F = \mu = m_t$ and $\mu_{0F} = \mu_0 = m_b$.

non-perturbative parameters α, β, δ and ϵ, accordingly set by the fitting procedure. This hadron-level similarity does however not lessen the importance of the higher degree of reliability of the perturbative-level calculation provided for by the soft-gluon resummation: Such an accurate calculation can be used as a firmer starting point for testing and fitting non-perturbative models. On the other hand, the inclusion of perturbative resummation cannot be expected to improve the agreement between hadron-level results obtained with different phenomenological non-perturbative ansëtze.

An alternative, and probably better, way of determining and including non-perturbative information makes use of moment space perturbative predictions and data [10]. The full hadron-level result can be written in N-space as the product of a perturbative and a non-perturbative contribution, $\Gamma_N^B = \Gamma_N^b D_N^{np}$. For each value of N one can then extract the corresponding D_N^{np} value from e^+e^- data, with no reference whatsoever to a specific hadronization model, and use it to predict the same moment in top decay. The DELPHI Collaboration [29] has recently published preliminary results for the moments of B-meson fragmentation in e^+e^- collisions up to $N = 5$. From these data, and using the moments of the e^+e^- perturbative contributions [6], one can extract D_N^{np}. The corresponding Γ_N^B values can then be calculated making use of the results for Γ_N^b obtained in this paper. Calling σ_N^B and σ_N^b the moments for the production rate of B mesons (measured) and b quarks (calculated in perturbative QCD) in e^+e^- annihilation, we have $\sigma_N^B = \sigma_N^b D_N^{np}$ and hence

$$\Gamma_N^B = \Gamma_N^b D_N^{np} = \Gamma_N^b \frac{\sigma_N^B}{\sigma_N^b} \,. \tag{5.6}$$

	$\langle x\rangle$	$\langle x^2\rangle$	$\langle x^3\rangle$	$\langle x^4\rangle$
e^+e^- data σ_N^B	0.7153±0.0052	0.5401±0.0064	0.4236±0.0065	0.3406±0.0064
e^+e^- NLL σ_N^b [A]	0.7666	0.6239	0.5246	0.4502
e^+e^- NLL σ_N^b [B]	0.7801	0.6436	0.5479	0.4755
D_N^{np} [A]	0.9331	0.8657	0.8075	0.7566
D_N^{np} [B]	0.9169	0.8392	0.7731	0.7163
t-decay NLL Γ_N^b [A]	0.7750	0.6417	0.5498	0.4807
t-decay NLL Γ_N^b [B]	0.7884	0.6617	0.5737	0.5072
t-decay Γ_N^B [A]	0.7231	0.5555	0.4440	0.3637
t-decay Γ_N^B [B]	0.7228	0.5553	0.4435	0.3633

Table 2: Experimental data for the moments σ_N^B from DELPHI [29], the resummed e^+e^- perturbative calculations for σ_N^b [6], the extracted non-perturbative contribution D_N^{np}. Using the perturbative results Γ_N^b, a prediction for the physical observable moments Γ_N^B is given. Set [A]: $\Lambda^{(5)} = 0.226\,\mathrm{GeV}$ and $m_b = 4.75\,\mathrm{GeV}$, set [B]: $\Lambda^{(5)} = 0.2\,\mathrm{GeV}$ and $m_b = 5\,\mathrm{GeV}$. The experimental error should of course be propagated to the final prediction.

Table 2 shows a practical implementation of this procedure. Predictions for the moments Γ_N^B of B-meson spectra in top decay are given, making use of the DELPHI experimental data. Two sets of perturbative results ([A] and [B]) are shown, the first using $\Lambda^{(5)} = 0.226\,\mathrm{GeV}$ and $m_b = 4.75\,\mathrm{GeV}$, the second using the default parameters of this paper. As expected, the perturbative calculations and the corresponding non-perturbative components differ, but the final predictions for the physical results Γ_N^B are to a large extent identical.

6. Conclusions

We have discussed soft-gluon resummation in top quark decay within the framework of perturbative fragmentation which, by making use of the DGLAP evolution equations, allows one to resum with NLL accuracy the large logarithms $\alpha_S \ln(m_t^2/m_b^2)$ which appear in the $\mathcal{O}(\alpha_S)$ massive calculation. The $\overline{\mathrm{MS}}$ coefficient function and the initial condition of the perturbative fragmentation function contain terms which become arbitrarily large for soft-gluon radiation. Such soft terms are process dependent in the coefficient function and process independent in the initial condition of the perturbative fragmentation function.

We have performed the resummation of soft-gluon effects in the coefficient function of the top-to-massless-quark decay process with NLL accuracy, and matched the resummed result to the full $\mathcal{O}(\alpha_S)$ one. This result has then been combined with the one of ref. [6], which resums NLL process-independent soft-gluon contributions in the initial condition of the perturbative fragmentation function, to produce a resummed prediction for the full top-to-massive-bottom decay.

We have presented the resummed b-quark energy spectrum in top-quark decay and compared it with the unresummed prediction. We have found that at large x_b the implementation of soft-gluon resummation has a visible impact: The x_b spectrum is smoothed

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out and shows the characteristic Sudakov peak. Our prediction for the b-energy spectrum is negative for $x_b \to 0$ and $x_b \to 1$; we have interpreted the behaviour at small x_b as due to unresummed $\ln x_b$ contributions in the coefficient function and at large x_b to missing non-perturbative effects.

We have investigated how the b-energy spectrum varies if we choose different values for the factorization and renormalization scales which enter our calculation. We have found that after including NLL soft-gluon resummation the distributions exhibit very little dependence on the choice of such scales, which corresponds to a reduction of the theoretical uncertainty of our prediction.

Finally, we have made use of b-flavoured hadron data in e^+e^- collisions to extract a non-perturbative contribution, and we used it to calculate hadron-level predictions in the top decay process. This has been done both in x_B space, tuning various hadronization models to e^+e^- distributions, and in N space, directly extracting the values of a few moments of the non-perturbative fragmentation function. In both cases care has been taken to make the procedure self-consistent by employing the same perturbative description in both the e^+e^- and top decay processes.

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